

$$(27) \log_{343} 7 = x$$

$$343^x = 7$$

$$x = \frac{1}{3}$$

$$(30) \log_6 \frac{1}{216} = x$$

$$6^x = \frac{1}{216}$$

$$x = -3$$

Practice: solve for x:

$$(1) \log_4 \frac{1}{16} = x$$

$$4^x = \frac{1}{16} \quad 4^{-2}$$

$$x = -2$$

$$(2) \log_{125} 5 = x$$

$$125^x = 5$$

$$x = \frac{1}{3}$$

$$125^{\frac{1}{3}} = \sqrt[3]{125}$$

$$(3) \log_2 \frac{1}{32} = x$$

$$2^x = \frac{1}{32}$$

$$x = -5$$

$$(4) \log_{64} 8 = x$$

$$64^x = 8$$

$$x = \frac{1}{2}$$

$$8^{2x} = 8^1$$

$$2x = 1$$

$$x = \frac{1}{2}$$

7.4 Properties of Logarithmic Functions

Obj: Students will be able to use the properties of logarithms to simplify expressions.

	<u>Exponents</u>	<u>Logarithms</u>
Product Property	$(a^m)(a^n) = a^{m+n}$ $x^3 \cdot x^4 = x^7$	$\log_b(m \cdot n) = \log_b m + \log_b n$ $\log_2 4 \cdot x = \log_2 4 + \log_2 x$
Quotient Property	$\frac{a^m}{a^n} = a^{m-n}$ $\frac{x^4}{x^3} = x^1$	$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$
Power Property	$(a^m)^n = a^{m \cdot n}$ $(x^4)^3$	$\log_b m^p = p \log_b m$ ie. $\log_b(a^4) = \log_b(a \cdot a \cdot a \cdot a)$ $= \log_b a + \log_b a + \log_b a + \log_b a$ $= 4 \log_b a$

Example 1: Write each logarithm as a single logarithm. write log once

1.) $\log_3 9 \oplus \log_3 3 =$

$$\log_3 9 \cdot 3 = \log_3 27$$

2.) $\log_2 5 \oplus \log_2 8 =$

$$\log_2 5 \cdot 8 = \log_2 40$$

3.) $\log_5 625 \ominus \log_5 125 =$

$$\log_5 \frac{625}{125} = \log_5 5$$

4.) $\log_7 20 \ominus \log_7 2 =$

$$\log_7 \frac{20}{2} = \log_7 10$$

5.) $\log_8 16 + \log_8 10 =$

$$\log_8 16 \cdot 10 = \log_8 160$$

6.) $\log_3 15 \ominus \log_3 5 =$

$$\log_3 \frac{15}{5} = \log_3 3$$

7.) $2\log_3 2 + 3\log_3 4 =$

$$\log_3 2^2 + \log_3 4^3$$

$$\log_3 4 \oplus \log_3 64$$

$$\log_3 4 \cdot 64 = \log_3 256$$

8.) $5\log_6 x - 3\log_6 y =$

$$\log_6 x^5 - \log_6 y^3$$

$$\log_6 \frac{x^5}{y^3}$$

Properties of Logarithms

Properties of Logarithms		
Product Property	Quotient Property	Power Property
$\log_b mn =$ _____	$\log_b \frac{m}{n} =$ _____	$\log_b m^n =$ _____

Write each expression as a one log single logarithm.

1. $\log_3 9 + \log_3 24$

$$\log_3 9 \cdot 24$$

$$\log_3 216$$

2. $\log_4 16^3$

$$\log_4 4096$$

3. $\log_2 7 - \log_2 9$

$$\log_2 \frac{7}{9}$$

4. $\log_3 8^5$

$$\log_3 32,768$$

5. $\log_4 x - \log_4 y$

$$\log_4 \frac{x}{y}$$

6. $\log 5 + \log 7$

$$\log 35$$

Expand each logarithm. Simplify if possible.

7. $\log_3 27x$

8. $\log \frac{3}{7}$

9. $\log_4 y^2 z^3$

10. $\log_5 \frac{3^2}{x}$

11. $\log_3 15xy$

12. $\log 8xz^4$

Properties of Logarithms

Write each expression as a single logarithm.

1. $\log_5 4 + \log_5 3$

2. $\log_6 25 - \log_6 5$

3. $\log_2 4 + \log_2 2 + \log_2 8$

$$\log_2 \frac{4 \cdot 2}{8}$$

$$\log_2 \frac{8}{8} = \log_2 1$$

4. $5 \log_7 x = 2 \log_7 x$

$$\log_7 x^5 = \log_7 x^2$$

5. $\log_4 60 - \log_4 4 + \log_4 x$

$$\log_4 \frac{60}{4x}$$

$$\log_4 \frac{15}{x}$$

6. $\log 7 - \log 3 + \log 6$

$$\log \frac{7}{3 \cdot 6}$$

$$\log \frac{7}{18}$$

7. $2 \log x - 3 \log y$

8. $\frac{1}{2} \log r + \frac{1}{3} \log s - \frac{1}{4} \log t$

9. $\log_3 4x + 2 \log_3 5y$

10. $5 \log 2 - 2 \log 2$

11. $\frac{1}{3} \log 3x + \frac{2}{3} \log 3x$

12. $2 \log 4 + \log 2 + \log 2$

13. $(\log 3 - \log 4) - \log 2$

14. $5 \log x + 3 \log x^2$

15. $\log_6 3 - \log_6 6$

Expand each logarithm. Simplify if possible.

22. $\log xyz$

23. $\log_2 \frac{x}{yz}$

24. $\log 6x^3y$

25. $\log 7(3x-2)^2$

26. $\log \sqrt{\frac{2rst}{5w}}$

27. $\log \frac{5x}{4y}$

28. $\log_5 5x^{-5}$

29. $\log \frac{2x^2y}{3k^3}$

30. $\log_4 (3xyz)^2$

Use the properties of logarithms to evaluate each expression.

44. $\log_2 8 + \log_2 4$

45. $\log_2 160 - \log_2 5$

46. $\log_6 27 + \log_6 8$

47. $\log_7 14 - \log_7 2$

48. $\log_4 64 + 2\log_4 2$

49. $\frac{1}{4}\log_3 162 - \log_3 \sqrt[4]{2}$

Properties of Logarithms

Expand each logarithm.

1) $\log (6 \cdot 11)$

2) $\log (5 \cdot 3)$

3) $\log \left(\frac{6}{11} \right)^5$

4) $\log (3 \cdot 2^3)$

5) $\log \frac{2^4}{5}$

6) $\log \left(\frac{6}{5} \right)^6$

$$7) \log \frac{x}{y^6}$$

$$8) \log (a \cdot b)^2$$

$$9) \log \frac{u^4}{v}$$

$$10) \log \frac{x}{y^5}$$

$$11) \log \sqrt[3]{x \cdot y \cdot z}$$

$$12) \log (x \cdot y \cdot z^2)$$

Condense each expression to a single logarithm.

$$13) \log 3 - \log 8$$

$$14) \frac{\log 6}{3}$$

$$15) 4 \log 3 - 4 \log 8$$

$$16) \log 2 + \log 11 + \log 7$$

$$17) \log 7 - 2 \log 12$$

$$18) \frac{2 \log 7}{3}$$

$$19) 6 \log_3 u + 6 \log_3 v$$

$$20) \ln x - 4 \ln y$$

$$21) \log_4 u - 6\log_4 v$$

$$22) \log_3 u - 5\log_3 v$$

$$23) 20\log_6 u + 5\log_6 v$$

$$24) 4\log_3 u - 20\log_3 v$$

Critical thinking questions:

$$25) 2(\log 2x - \log y) - (\log 3 + 2\log 5)$$

$$26) \log x \cdot \log 2$$

7.4 continued...

Obj: Students will be able to use the properties of logarithms to simplify expressions.

Exponential-Logarithmic

$$\log_b b^x = x$$

Inverse Properties

$$b^{\log_b x} = x$$

Example 1: Evaluate each expression.

$$a.) \log_5 5^4 =$$

$$b.) 3^{\log_3 9} =$$

You try:

1.) $\log_6 6^3 =$

2.) $5^{\log_5 13} =$

3.) $12^{\log_{12} 5} =$

4.) $\log_{10} 10^8 =$

